Modeling coherent Doppler lidar sensing of turbulent atmosphere

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ABSTRACT

The influence is simulated of the atmospheric turbulent motion on the character and the accuracy of the radial velocity profiles recovered by using a high-range-resolution coherent-Doppler-lidar approach we have developed recently. The simulations are based on an original statistical spatio-temporal model of the turbulent radial-velocity fluctuations supposed to have a von-Kàrmàn-like spectrum that closely approximates the well-known Kolmogorov-Obukhov spectrum. The results from the simulations confirm the basic conclusions we have analytically deduced formerly about the character of the recovered radial velocity profiles, depending on the duration of the lidar measurement procedure. In this way the model adequacy is also substantiated.

Keywords: Coherent Doppler lidar, coherent heterodyne detection, turbulent atmosphere

1.INTRODUCTION

Recently we have developed a novel coherent Doppler lidar approach for recovering with high range resolution of radial (Doppler) velocity profiles in the atmosphere [1-4]. The approach concerned is based on an analysis of the autocovariance of the complex heterodyne lidar signal and allows one to achieve a resolution interval below the sensing laser pulse length. Averaging over some sufficient number N of conjecturally independent signal realizations (obtained by N laser shots) is a way to obtain accurate autocovariance estimates. Such a measuring procedure would require however a too long observation (data accumulation, measurement) time, e.g. of the order of many minutes, exceeding not only all the correlation scales of the signal but also the period of stationarity of the atmosphere. The contemporary powerful-enough pulsed-laser transmitters for coherent lidars can have a pulse repetition rate ~ 1kHz[5]. Thus, for a few seconds they can provide a sufficiently large number of signal realizations to average the small-temporal-scale signal fluctuations, due e.g. to the reflective-speckle and refractive-turbulence effects, and to the weak fast-varying turbulent velocity fluctuations. At the same time the stronger, slowly varying turbulent velocity fluctuations, whose correlation scales exceed the observation time, may not be averaged. Then the estimate obtained of the signal autocovariance could be considered as containing an average, for the observation period, Doppler velocity profile. This profile should coincide with the parent-population mean Doppler velocity profile (under stationary atmospheric conditions) when the measurement time exceeds essentially the mean velocity-fluctuation correlation time; on the contrary, it should nearly coincide with the instantaneous radial velocity profile when the measurement time is much shorter than the mean correlation time. Such a physical interpretation of the profiles to be retrieved has initially been given in Ref.3, and then analytically substantiated in Ref.4. A purpose of the present work is to verify by simulations the above-described conception about the profiles to be retrieved. For this purpose an original statistical model has been developed and used for the spatio-temporal turbulent fluctuations of the radial velocity in the atmosphere. Since this model itself may be of independent interest and use for other atmospheric studies, another aim of the work is to describe it briefly and to prove its efficiency in the simulation process.

2. HIGH-RESOLUTION RETRIEVING OF DOPPLER-VELOCITY PROFILES

We shall consider here only the velocity-fluctuation effects and, certainly, the naturally present reflective-speckle effects. Then the model of the coherent lidar signal is expressible in the following simplified form (see also [1,2]):

$$I(t = 2z/c) = \int_{z_0}^{ct/2} [f(t - 2z/c)]^{1/2} \exp[j\omega_m(z)t]dA(z) \ , \tag{1}$$

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where $I(t{=}2z/c)$ is the complex heterodyne lidar signal at the instant t (after the pulse emission) corresponding to the pulse front position z along the lidar line of sight, j is imaginary unity; $[0, z_0]$ is the lidar blind zone; $f(\vartheta)$ describes the sensing pulse power shape normalized to its peak value; $\omega_m(z) = \omega_0 \chi(z) - \omega_h$ is the intermediate frequency, ω_0 is the frequency of the sensing radiation, ω_h is the local-oscillator frequency, $\chi(z) = 1 - 2V(z)/c$, $V(z)$ is the radial velocity profile, c is the speed of light; $dA(z)$ is a differential circular complex Gaussian random quantity such that $< dA(z)dA(z') >= \Phi(z)\delta(z - z')dzdz'$, $<>$ and δ denote respectively ensemble average and delta function, and $\Phi(z)$ describes [1-3] the maximum-resolved signal power profile obtainable at sufficiently short laser pulses (physical δ - pulses).

The signal autocovariance $Cov(t,\theta) =< I^*(t)I(t+\theta) >$ is obtainable from Eq. (1) in the form

$$Cov(t,\theta) = \int_{z_0}^{ct/2} dz\left[f(t - 2z/c)f(t+\theta-2z/c)\right]^{1/2}\Phi(z) < \exp[j\omega_m(z)\theta] > \ , \tag{2}$$

where $< \exp[j\omega_m(z)\theta] >$ is an ensemble average over the realizations $\{V(z)\}$ of the Doppler velocity profile; * denotes complex conjugation. An estimate $C\hat{o}v(t,\theta)$ of the autocovariance has usually been obtained on the basis of the relation

$$C\hat{o}v(t,\theta) = N^{-1}\sum_{k=1}^{N} I_k{}^*(t)I_k(t+\theta) \ , \tag{3}$$

where $I_k(t)$ (k=1,2,...,N) are N signal realizations obtained by N laser shots. Assuming that only the reflective-speckle-due signal fluctuations [described by $dA(z)$] are entirely averaged, Eq.(3) can be rewritten as [see also Eqs.(1) and (2)]:

$$C\hat{o}v(t,\theta) = \int_{z_0}^{ct/2} dz\left[f(t - 2z/c)f(t+\theta-2z/c)\right]^{1/2}\Phi(z)\overline{\exp[j\omega_m(z)\theta]} \ , \tag{4}$$

where $\overline{\exp[j\omega_m(z)\theta]} = N^{-1}\sum_{k=1}^{N}\exp[j\omega_{mk}(z)\theta]$, $\omega_{mk}(z) = \omega_0\chi_k(z) - \omega_h$, $\chi_k(z) = 1 - 2V_k(z)/c$, and $V_k(z)$ is the realization of the radial-velocity profile at the k-th laser shot. The solution obtained of Eq.(2) with respect to $\omega_m(z = ct/2)$ is expressible as

$$\omega_m(z = ct/2) = [\pi c\Phi(z)]^{-1}\int_{-\infty}^{-\infty} d\Omega[\tilde{R}(\Omega)/\tilde{f}(\Omega)]\exp(-j\Omega t) \ , \tag{5}$$

or

$$\omega_m(z = ct/2) = H(z = ct/2)/\Phi(z = ct/2) \ , \tag{6}$$

where $\tilde{f}(\Omega)$ and $\tilde{R}(\Omega)$ are the Fourier transforms of functions $f(\vartheta)$ and $R(\vartheta) = \operatorname{Im} Cov^I(\vartheta, \theta = 0)$, ^{I} denotes first derivative with respect to θ (taken at θ =0), and $H(z = ct/2)$ is the unique continuous solution of the following (Volterra) integral equation

$$R(t = 2z/c) = \int_{z_0}^{ct/2} f(t - 2z'/c)H(z')dz' \ . \tag{7}$$

Based on Eq.(4), $\omega_m(z)$ is deciphered to be equal in general to the arithmetic-mean intermediate-frequency profile

$$\omega_m(z) = N^{-1}\sum_{k=1}^{N}\omega_{mk}(z) = \omega_0[1 - (2/c)N^{-1}\sum_{k=1}^{N}V_k(z)] - \omega_h \ , \tag{8}$$

that may be quite different from the ensemble-mean profile $< \omega_m(z) >= \omega_0[1 - 2 < V(z) > /c] - \omega_h$; $\omega_m(z)$ is an estimate of the latter only when the measurement time T exceeds essentially the velocity fluctuation correlation time t_c (see e.g. [4]). In the opposite case ($T<<t_c$) $\omega_m(z)$ describes, in practice, an instantaneous intermediate-frequency (radial-velocity) profile $\omega_m(z)[V(z)]$.

3. SPATIO-TEMPORAL STATISTICAL MODEL OF THE TURBULENT RADIAL-VELOCITY FLUCTUATIONS

For simulating the realizations $V_k(z)$ of the radial-velocity profile one should create an adequate spatio-temporal statistical model of the radial-velocity fluctuations $\widetilde{V}(z,\vartheta)=V(z,\vartheta)-<V(z,\vartheta)>$, taking into account the correlation properties of the turbulent motion and the computer spatial and temporal sampling, with intervals Δz and $\Delta\vartheta$, respectively. This task is solved here through an appropriate linear filtration of a spatio-temporal normally-distributed unitary-variance discrete white noise $w_d(r=l\Delta r, \vartheta=m\Delta\vartheta)\equiv w_d(l,m)$ $(l,m=\pm1,\pm2,...)$ representing the discrete samples of a spectrally-limited continuous white noise with boundary wave-number $K_b=\pi/\Delta z$ and boundary frequency $\omega_b=\pi/\Delta\vartheta$. The resultant radial-velocity fluctuation field $\widetilde{V}(l,m)$ is then obtained as

$$\widetilde{V}(l,m)=\sum_{n,s=-\infty}^{\infty}C_{ns}w_d(l-n,m-s)\ , \tag{9}$$

where C_{ns} are the elements (filtering coefficients) of the infinite filtering matrix $C=\{C_{ns}\}$. An explicit expression of C_{ns} we have derived and used in the simulations is

$$C_{ns}=\sigma_v c_{ns}=\sigma_v\pi^{-7/4}(\xi\Delta z\Delta\vartheta\tau_l/16)^{1/2}\int_{-\pi/\Delta z}^{\pi/\Delta z}dK\int_{-\pi/\Delta\vartheta}^{\pi/\Delta\vartheta}d\omega\exp[j(\omega s\Delta\vartheta-Kn\Delta z)(K^2+K_0^2)^{-5/12}$$
$$\times\ \exp\{-K^2/(2K_m^2)-(\omega-KV_T)^2\tau_l^2/8\}\ , \tag{10}$$

where $\xi=\pi/\int_0^{\infty}dK(K^2+K_0^2)^{-5/6}\exp(-K^2/K_m^2)$, σ_v^2 is the variance of the radial-velocity fluctuations, V_T and τ_l are some mean translational velocity and lifetime, respectively, of the radial-velocity inhomogeneities, $K_0=1/L_0$, L_0 is the outer turbulence scale, $K_m=5.92/l_m$, $l_m=l_0(12C^2)^{3/4}$, l_0 is the inner turbulence scale, and $C^2=1.77$. The derivation of Eq.(10) is based on a von-Kàrmàn-like approximation of the spatial spectrum (and corresponding correlation or structure functions) of the Doppler velocity fluctuations that closely approximates the well-known Kolmogorov-Obukhov spectrum [2]. A property of the coeficients c_{ns} that is useful for numerical tests is that $\sum_{n,s=-\infty}^{\infty}c_{ns}^2=1$.

4. SIMULATIONS

The complex heterodyne lidar signal $I(t=2z/c)$ is synthesized according to the following discrete version of Eq.(1):

$$I(t=2z/c)=\sum_{l=l_1+1}^{l_2}[f(t-2z_l/c)]^{1/2}\exp[j\omega_m(z_l)t][\Phi(z_l)\Delta z]^{1/2}w_l\ , \tag{11}$$

where the (measurable) pulse shape $f(\vartheta)$ is taken in the form represented in Fig.1 (inset), $[\Phi(z_l)\Delta z]^{1/2}w_l$ corresponds to $dA(z_l)$, the profile employed of $\Phi(z)$ is given in Fig.1, $w_l\equiv w=w_r+jw_i$ is a circular complex Gaussian random quantity with zero mean value $<w>=<w_r>=<w_i>=0$ and unitary variance $Dw=<|w|^2>=<w_r^2>+<w_i^2>=1$ $(<w_r^2>=<w_i^2>=1/2)$, $<w_r\ w_i>=0$ and $<w_l\ w_{l+s}>=0$ for $S\neq0$, $l_1=z_0/\Delta z$, $l_2=ct/(2\Delta z)$, and $z_l=l\Delta z$. The realizations $\omega_{mk}(z)=\omega_0[1-2V_k(z)/c]-\omega_k$ of the intermediate-frequency profile $\omega_m(z)$ $[\omega_m(z_l)]$ are modeled statistically by considering each realization $V_k(z)$ of the Doppler velocity profile as a sum $V_k(z)=V_m(z)+\widetilde{V}_k(z)$ of the ensemble-mean Doppler velocity profile $V_m(z)=<V(z)>$ and the profile of the turbulent velocity fluctuations $\widetilde{V}_k(z)$. The profile of $V_m(z)$ used in the simulations is shown in Fig. 3. Let us note that the temporal (spatial) variation scales of both the profiles, $\Phi(z)$ and $V_m(z)$, are chosen to be much smaller than the pulse duration (pulse length). The spatio-temporal velocity fluctuation random field $\{\widetilde{V}(z,\vartheta)\}$ is built according to the procedure described in Sec.3 with $\Delta z=1.5$ m, $\Delta\vartheta=0.01$ s, $\sigma_v=4$ m/s, $\tau_l=5$ s, $V_T=5$ m/s, $L_0=20$m, and $l_0=0.001$ m. Thus, the velocity –fluctuation correlation time is $t_c \sim \min[\tau_l, L_0 / V_T] \sim 4-5s$. The sum of the squared coefficients c_{ns} (see Sec.3) is numerically proven to be equal to unity. Also, the normalized covariance $K_v(Z,\theta) = <\tilde{V}(z+Z,\vartheta+\theta)\tilde{V}(z,\vartheta)> / <\tilde{V}^2>$ of the spatio-temporal radial-velocity fluctuations $\tilde{V}(z,\vartheta)$ is shown to closely coincide with that given preliminarily (see Fig.2). The realizations $V_k(z)$ of the Doppler velocity profile are then extracted from $\{\tilde{V}(z,\vartheta)\}$ as $V_k(z) = \tilde{V}[z,(k-1)q\Delta\vartheta]$, where q is an integer, and $q\Delta\vartheta$ is the interval between the adjacent laser shots. Taking into account in Eq. (11) the profiles chosen of $f(\vartheta)$, $\Phi(z)$ and $V_m(z)$ as well as the realizations built w_k of w, and $\tilde{V}_k(z)$ of $\tilde{V}(z)$, we obtain the corresponding signal realizations $I_k(t=2z/c)$ at N successive laser shots. Given $I_k(t=2z/c)$, the signal autocovariance estimate $C\hat{o}v(t,\theta)$ is obtained on the basis of Eq.(3). An estimate $\hat{\Phi}(z)$ of the profile $\Phi(z)$ is further obtained by deconvolution techniques [6]. At last, the recovered Doppler velocity profiles $V_r(z)$ are obtained on the basis of the retrieval algorithms [Eqs.(5) and (6)] described in Sec.2. Let us note that, to avoid the necessity of too large number of laser shots to average the small-scale signal fluctuations, the estimates $C\hat{o}v(t,\theta)$, $\hat{\Phi}(z)$ and $V_r(z)$ are filtered by a monotone smooth sharp-cutoff digital filter with $7\Delta\vartheta$-wide window [7,8] that is much narrower than the variation scale of $V(z)$.

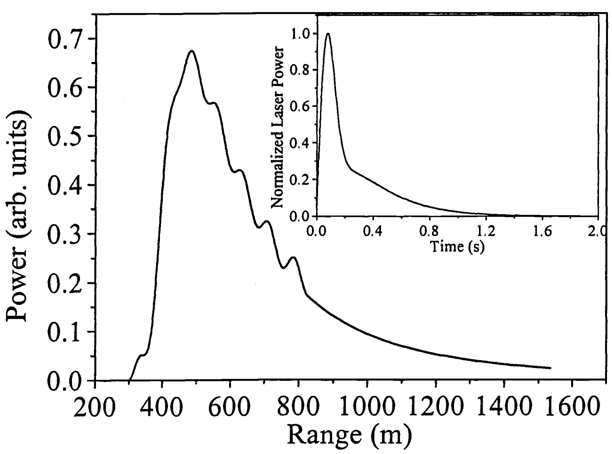


Fig.1. Models of the maximum-resolved signal power profile $\Phi(z)$ and the pulse shape $f(\vartheta)$ (inset) used in the simulations.

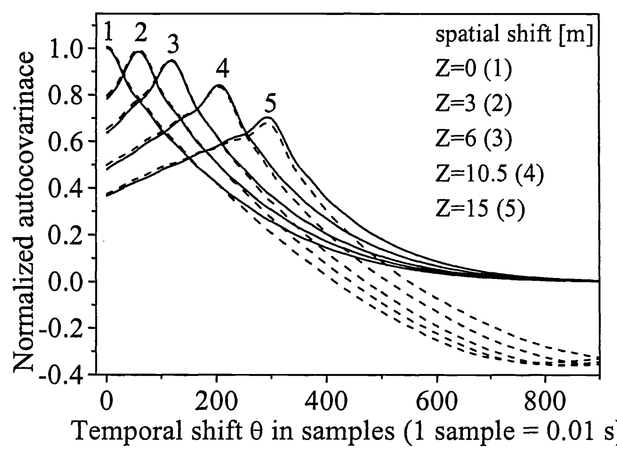
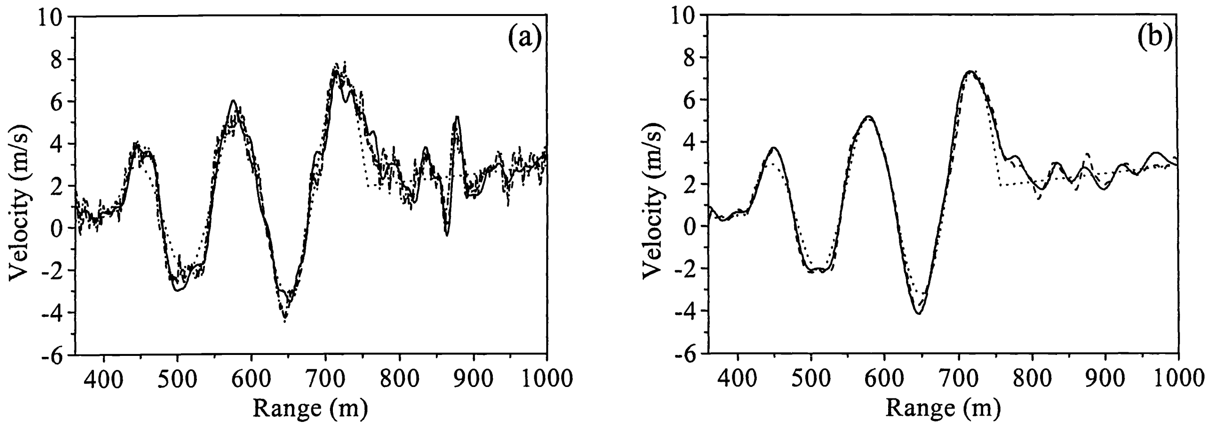


Fig.2. Velocity-fluctuation normalized autocovariance: model (solid curves) and its reproduction by simulations (dashed curves).

The short-term ($T<<t_c$), middle-term ($T\sim t_c$), and long-term ($T>>t_c$) lidar measurement procedures are simulated by appropriate choice of the interval $q\Delta\vartheta$ (see above). So, we have simulated, respectively, N = 300, 200, and 140 laser shots produced within measurement intervals T = 1.2 s, 4 s, and 280 s. The corresponding restored Doppler velocity profiles $V_r(z)$ are shown together and compared, in Figs.3a, 3b, and 3c, with the arithmetic-mean profile $V_a(z) = N^{-1}\sum_{k=1}^{N} V_k(z)$, the ensemble-mean profile $V_m(z) = \langle V(z) \rangle$, and the profile $V_1(z)$ at only one (say the first) laser shot. As it is seen in the figures, the results from the simulations confirm the previsions about the character of the recovered profiles. So, in all the cases the profile $V_r(z)$ closely fits $V_a(z)$ [see Eq.(8)]. Also, in the case of short-term measurement (Fig.3a) the profile $V_r(z)$, being in general different from $V_m(z)$, is near each profile $V_k(z)$. At a middle-term measurement procedure (Fig.3b) the profile $V_r(z)$ may differ from both $V_m(z)$ and $V_k(z)$. At last, at a long-term measurement (Fig.3c) the restored profile $V_r(z)$ may differ from $V_k(z)$, but nearly approximates $V_m(z)$ because, according to the law of averages, $V_a(z)$ [$\sim V_r(z)$] should tend to $V_m(z)$ when $N > T/t_c >> 1$.



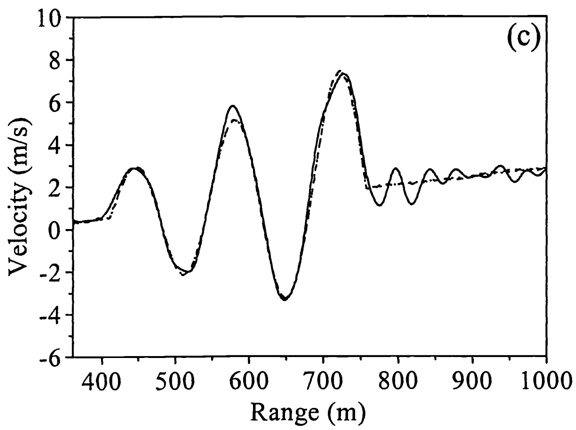


Fig.3. Comparison of restored Doppler velocity profiles $V_r(z)$ (solid curves) with the ensemble-mean profiles $V_m(z)$ (dotted curves), the arithmetic-mean profiles $V_a(z)$ (dashed curves) and the instantaneous profile $V_1(z)$ (dashed-dotted curve, a) in the cases of short-term (a), middle-term (b) and long-term (c) measurement procedures.

5.CONCLUSION

The statistical modeling and the simulations performed in the work confirm the physical interpretation [3] and the analytical conclusions [4] concerning the character of the Doppler velocity profiles recovered on the basis of an analysis of the autocovariance of the coherent Doppler lidar signal. That is, a short-term, long-term, or middle-term measurement procedure allows one to determine, respectively, a near instantaneous, the ensemble-mean, or the arithmetic-mean Doppler velocity profile. Also, the results from the simulations illustrate the adequacy and the efficiency of the developed statistical model of the turbulent radial-velocity fluctuations.

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REFERENCES

1. L. L. Gurdev, T. N. Dreischuh, and D. V. Stoyanov, J. Opt. Soc. Am. A **18**, pp.134-142, 2001.
2. L.L. Gurdev, T.N. Dreischuh, and D.V. Stoyanov, Appl. Opt. **41**, pp.1741-1749, 2002.
3. L.Gurdev, T.Dreischuh, Optics Communications **219**, pp.101-116, 2003.
4. Ljuan L. Gurdev, Tanja N. Dreischuh, Proc. SPIE **5226,** pp.300-304, 2003.
5. G.N.Pearson, Rev. Sci. Instum. **64**, pp.1155-1157, 1993.
6. L.L.Gurdev, T.N.Dreischuh, and D.V.Stoyanov, J. Opt. Soc. Am. A**10**, pp.2296-2306, 1993.
7. R.W.Hamming, *Digital filters*, Prentice Hall, Englewood Cliffs, N.J., 1983.
8. T.Dreischuh, L.Gurdev, D.Stoyanov, Balkan Physics Letters (Proc. Suppl.) BPU-4 (2000), pp.59-62.