An heuristic view on the signal-to-noise ratio at coherent heterodyne detection of aerosol lidar returns formed through turbulent atmosphere

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ABSTRACT

The main purpose of the study is to estimate the root-mean-square signal-to-noise ratio (*rmsSNR*) characterizing the coherent heterodyne detection of aerosol-backscattered lidar returns influenced by the turbulent fluctuations of the refractive index in the atmosphere. A general expression is obtained heuristically that describes the *rmsSNR* as depending on (generally turbulence-affected) the return-intensity relative variance and the ratio of the coherence area of the lidar return to the receiving-aperture area. On the basis of the expression obtained, the *rmsSNR* is estimated as a function of the distance (of the scattering volume) along the line of sight at different values of the wavelength of the sensing radiation, the receiving-aperture radius, the transmitted beam-pulse radius, and the refractive-index turbulent parameter C_n^2. It is shown that the *rmsSNR* values obtained at different distances under different experimental conditions are mostly near the unity, and the coherent heterodyne lidar signal should have correspondingly circular complex Gaussian statistics.

Keywords: Coherent heterodyne detection, coherent lidar, signal-to-noise ratio, turbulent atmosphere

1. INTRODUCTION

The *rmsSNR*[1] has often been missed when estimating the total signal-to-noise ratio (*total SNR*) of the coherent heterodyne detection of aerosol-backscattered lidar returns. The *rmsSNR* is however an essential characteristic of the noise in this case. It is the upper limit of the *total SNR* achievable when the mean heterodyne-signal power exceeds considerably the mean power of the additive measurement noise[2]. The contemporary (high pulse repetition rate) laser transmitters for coherent lidars are designed in such a way that to ensure powerful enough signals, in the above sense, from distances of several kilometers within the planetary boundary layer (see e.g. Ref.3). When the atmospheric refractive-turbulence effects are negligible the *rmsSNR* reaches its maximum value that is equal to unity. Thus, the actual single-shot *SNR* of the heterodyne detection of aerosol lidar returns cannot exceed unity. To increase the effective *SNR* one should certainly use some averaging and filtering procedures including multishot operation (e.g. [4]).

The reflective-speckle and refractive-turbulence effects would decrease essentially the coherence area of the lidar return, far below the receiving aperture area. Therefore one may expect that even under strong-turbulence conditions the complex heterodyne lidar signal would approximately behave as a circular complex Gaussian random quantity obtained by addition on an amplitude basis of many independent speckle patterns[1]. Then the *rmsSNR* should be always near the unity.

The main purpose of the study is to prove the above-described supposition, that is, to estimate the *rmsSNR* characterizing the coherent heterodyne detection of aerosol-backscattered lidar returns influenced by the turbulent fluctuations of the refractive index in the atmosphere. For this purpose a general expression is obtained heuristically describing the *rmsSNR* as depending on (generally turbulence-affected) the relative variance of the return intensity and the ratio of the coherence area of the lidar return to the receiving-aperture area. On the basis of the expression obtained, the *rmsSNR* is estimated as a function of the distance (of the scattering volume) along the lidar line of sight at different values of the wavelength of the sensing radiation, the receiving-aperture radius, the transmitted beam-pulse radius, and the refractive-index turbulent parameter C_n^2. It is shown that the *rmsSNR* values obtainable at different distances under different experimental conditions should be mostly near the unity for conventionally large receiving aperture areas. Correspondingly, the coherent heterodyne lidar signal should have nearly circular complex Gaussian statistics[5,6].

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2. *RMS SNR* AT COHERENT DETECTION OF LIDAR RETURNS FROM AEROSOLS

2.1. *Total SNR*

The *total SNR* at coherent detection of lidar returns has usually been defined[2] as the ratio of the mean heterodyne-signal power P_m to square root of the variance $VarP_t$ of the total (signal plus noise) power P_t at the detector output. For an additive, statistically-independent zero-mean circulo-complex Gaussian noise process, the expression of the *total SNR* has been derived in the form[2]

$$total\ SNR = \{(CNR/2)/[1+(CNR/2)/(rmsSNR)^2+(2\ CNR)^{-1}]\}^{1/2}\ , \tag{1}$$

where $CNR = P_m/P_N$ is the so-called carrier-to-noise ratio, P_N is the mean additive-noise power, $rmsSNR = P_m/(VarP)^{1/2}$ and $VarP$ is the variance of the heterodyne-signal power P. It is seen that the *total SNR* $\cong$ *rmsSNR* for $CNR >> 1$. That is, when the signal power exceeds essentially the additive-noise power the *total SNR* is determined mainly by the signal-amplitude fluctuations.

2.2. *rmsSNR*

Further we shall concentrate our attention on the analysis of the *rmsSNR* at coherent detection of lidar returns. For this purpose we shall first derive heuristically an expression describing its behaviour. An axially-symmetrical geometry of the problem will be implied throughout.

Let us suppose that the photodetector of the coherent lidar collects all the local-oscillator beam energy and all the backscattered radiation that is covered by the receiving optical system. Then the photomixing may be considered as taking place on the receiving aperture plane $\{\vec{\rho}\}$, and the local-oscillator field amplitude distribution $\vec{E}_h(\vec{\rho})$ will play the role of an aperture weighting function. Correspondingly, the local-oscillator beam radius R will play the role of an effective receiving-aperture radius. In general, the complex amplitude of the heterodyne-signal photocurrent $J(t)$ resulting from coherent detection of the lidar return (at the moment t after the emission of the sensing laser pulse) may be represented as[7]

$$J(t) = K_o \int \vec{E}_b(\vec{\rho},t)\vec{E}_h^*(\vec{\rho})d\vec{\rho}\ , \tag{2}$$

where $K_o = 2qe/\hbar\omega$, q is the effective quantum efficiency of the photodetector, e is the electron charge, $\hbar = h/2\pi$, h is the Plank's constant, $\omega = (\omega_o + \omega_h)/2$, ω_o and ω_h are respectively the circular frequencies of the sensing radiation and the local oscillator, $\vec{E}_b$ is the field amplitude vector of the lidar return and the superscript * denotes complex conjugation. Considering a fixed moment of detection $t = 2z/c$ (corresponding to a distance z of the scattering volume along the line of sight) and assuming a linear polarization of $\vec{E}_h$, we can further write $E_b(\vec{\rho})$ and $E_h(\vec{\rho})$, instead of $\vec{E}_b(\vec{\rho},t)$ and $\vec{E}_h(\vec{\rho},t)$, where E_h is the module of $\vec{E}_h$, and E_b is interpreted as the polarization component of $\vec{E}_b$ colinear with $\vec{E}_h$.

When the characteristic coherence radius ρ_c of the lidar return field $E_b(\vec{\rho})$ is much less than R, the mean signal power P_m is obtainable by means of Eq.(2) in the form

$$P_m = \langle JJ^* \rangle = \langle |J|^2 \rangle = K_o^2 \langle I \rangle \sigma(\rho_c, R) P_h\ , \tag{3}$$

where $\sigma(\rho_c,R)$ is a mean (over the aperture) coherence area, generally depending on ρ_c and R, $P_h = \int |E_h(\vec{\rho})|^2 d\vec{\rho}$ is the power of the local oscillator, $\langle I \rangle = \langle E_b(\vec{\rho})E_b^*(\vec{\rho}) \rangle = \langle |E_b|^2 \rangle$, and <.> and |.| denote, respectively, ensemble average and module. The same result can be obtained on the basis of an heuristic model considering the receiving aperture as consisting of many equal, statistically independent coherence areas $\sigma_i = \sigma(\rho_c,R)$ such that

$$J = K_o \sum_i E_{bi}E_{hi}^*\sigma_i = K_o\sigma(\rho_c,R)\sum_i E_{bi}E_{hi}^*\ , \tag{4}$$

where E_{bi} and E_{hi} are assumed to be approximately constant within each area σ_i. The simplified analysis based on Eq.(4) leads to the following expression of the mean square signal power $<P^2>$:

$$\langle P^2 \rangle = K_o^4\sigma^4 \left\{ \langle I^2 \rangle \sum_i I_{hi}^2 + 2\langle I \rangle^2 \sum_{k,l} I_{hk} I_{hl} \right\}\ , \tag{5}$$

where $I=|E_b|^2$, and $I_h=|E_h|^2$. On the basis of Eqs.(3) and (5), taking into account that in fact $P_m = K_o^2\sigma^2\langle I\rangle\sum_i I_{hi}$, we obtain the following expression of the relative variance $V_r = VarP/P_m^2 = \left(\langle P^2\rangle - P_m^2\right)/P_m^2 = (rmsSNR)^{-2}$ of the heterodyne signal power:

$$V_r = 1+(V_I-1)\left[\sum_i I_{hi}^2 / \left(\sum_i I_{hi}\right)^2\right] \quad , \tag{6}$$

where $V_I = \left(\langle I^2\rangle - \langle I\rangle^2\right)/\langle I\rangle^2$ is the relative variance of the intensity of the detected radiation.

When $\rho_c >> R$ (spatially coherent optical signals) it is evident that $V_r = V_I$.

Thus, one can write, in general, that

$$V_r = 1+(V_I - 1)\ \eta(\rho_c,R) \quad , \tag{7}$$

where $\eta(\rho_c,R)$ is a factor depending on the relation between ρ_c and R; $\eta = 1$ when $\rho_c >> R$, and $\eta = \sum_i I_{hi}^2/(\sum_i I_{hi})^2$ when $\rho_c << R$. In the latter case ($\rho_c << R$) the value of $\eta \sim \sigma/S \sim \rho_c^2/R^2$, where S is some effective receiving aperture area. According to Eq.(7), the *rmsSNR* is given by

$$rmsSNR = [1+(V_I - 1)\ \eta(\rho_c,R)]^{-1/2} \quad . \tag{8}$$

Eq.(8) shows that the heterodyne detection of typical speckle patterns ($\rho_c << R$ or $V_I \sim 1$) is characterized by a *rmsSNR* of the order of unity; there is in this case some type of aperture averaging of the possible "extraspeckle" intensity fluctuations characterized by the quantity V_I -1.

3. *RMS SNR* VARIATION ALONG THE LIDAR LINE OF SIGHT

3.1. Factor $\eta(\rho_c,R)$

For a colimated Gaussian sensing laser beam-pulse with radius a, one can deduce, on the basis of an approach employed e.g. in Ref.7, that

$$\eta(\rho_c,R) = \eta(k,C_n^2,z,a,R) = [1+2R^2/a^2+k^2R^2(a^2+R^2)/4z^2+4R^2/\rho_t^2]^{-1} \quad , \tag{9}$$

where $k=2\pi/\lambda$ is the wavenumber of the sensing radiation and λ is its wavelength, C_n^2 is the refractive-turbulence structure parameter, z is the distance along the line of sight, and $\rho_t = (1.46\ C_n^2\ k^2 z)^{-3/5}$ is the turbulence-conditioned coherence radius. The over-all mean coherence radius of the lidar return on the receiving aperture is $\rho_c = [R^{-2} + 2a^2 + k^2(a^2+R^2)\ /\ 4z^2 + 4\rho_t^{-2}]^{-1/2}$. It is seen [Eq.(9)] that $\eta \to 1$ for $\rho_c >> R$, and $\eta = \rho_c^2/R^2 = \sigma/S$ in general.

3.2. Relative intensity fluctuations of lidar returns from aerosol layers or diffuse targets.

According to some theoretical and experimental studies[8-10], the relative intensity variance V_I of turbulence-influenced lidar returns from incoherent scatterer ensembles (aerosol layers or diffuse targets) is practically always equal to unity. Correspondingly, the *rmsSNR* should also, in practice, be equal to unity. A possible interpretation of such a result is that, on the one hand, under weak turbulence conditions the lidar-return speckle pattern is not essentially affected by the fluctuations of the atmospheric refractive index; on the other hand, at high turbulence levels the lidar return consists of many independent contributions influenced by different turbulent whirls. To better understand the physical picture in this case, it is perhaps expediently, except a rigorous theory[10], to develop also a simplified but more viewable theory of the discussed phenomenon. Let us note that the results from other investigations[2,11] suggest that the variance V_I should increase with increasing the turbulence effects.

In general, V_I may have a more complicated behaviour. Therefore we shall further consider a rather hypothetical but more general case of a changeable variance V_I that depends on $\beta_o = 1.23\ C_n^2\ k^{7/6} z^{11/6}$. For $\beta_o < 1$ [$(z/k)^{1/2} < \rho_t$, a region of weak log-amplitude fluctuations] we assume that[2]

$$V_I = 2\exp(\beta_o^2) - 1 \quad . \tag{10a}$$

For $\beta_o > 1$ [$(z/k)^{1/2} > \rho_t$, a region of strong log-amplitude fluctuations] the expression of V_I is chosen to be[12]

$$V_I = 1 + 2\,[1 + 0.85(\beta_o^2)^{-2/5}] \quad , \tag{10b}$$

i.e. to describe the saturation of the relative intensity fluctuations. Such a choice of V_I corresponds to a physical situation when all elementary waves from different scatterers are modulated simultaneously by the same turbulent refractive-index fluctuations.

3.3. *rmsSNR* variation

The *rmsSNR* is calculated numerically, on the basis of Eqs.(7-10), as a function of z at different values of the parameters λ, R, a, and C_n^2 . The results obtained are illustrated in Figs. 1 and 2. It is seen that in the case of a conventional receiving aperture radius $R \sim 0.1$ m the *rmsSNR* is practically equal to unity, especially at higher turbulence levels C_n^2, shorter wavelengths λ, and narrower in general sensing laser beams. Such a result is due to the partial aperture averaging, discussed above, that is proper to the coherent heterodyne detection of lidar returns. In this case we have $\rho_c << R$ [$\eta << 1$, see Eq.(8)], independently of the distance z along the line of sight. Thus, the coherent lidar return signal is formed on the basis of many statistically independent contributions and should have a (near) circulo-complex Gaussian statistics[1]. For smaller radii R, e.g. $R \sim 0.01$ m, the partial aperture averaging effect is no of importance at all (see the Figures).

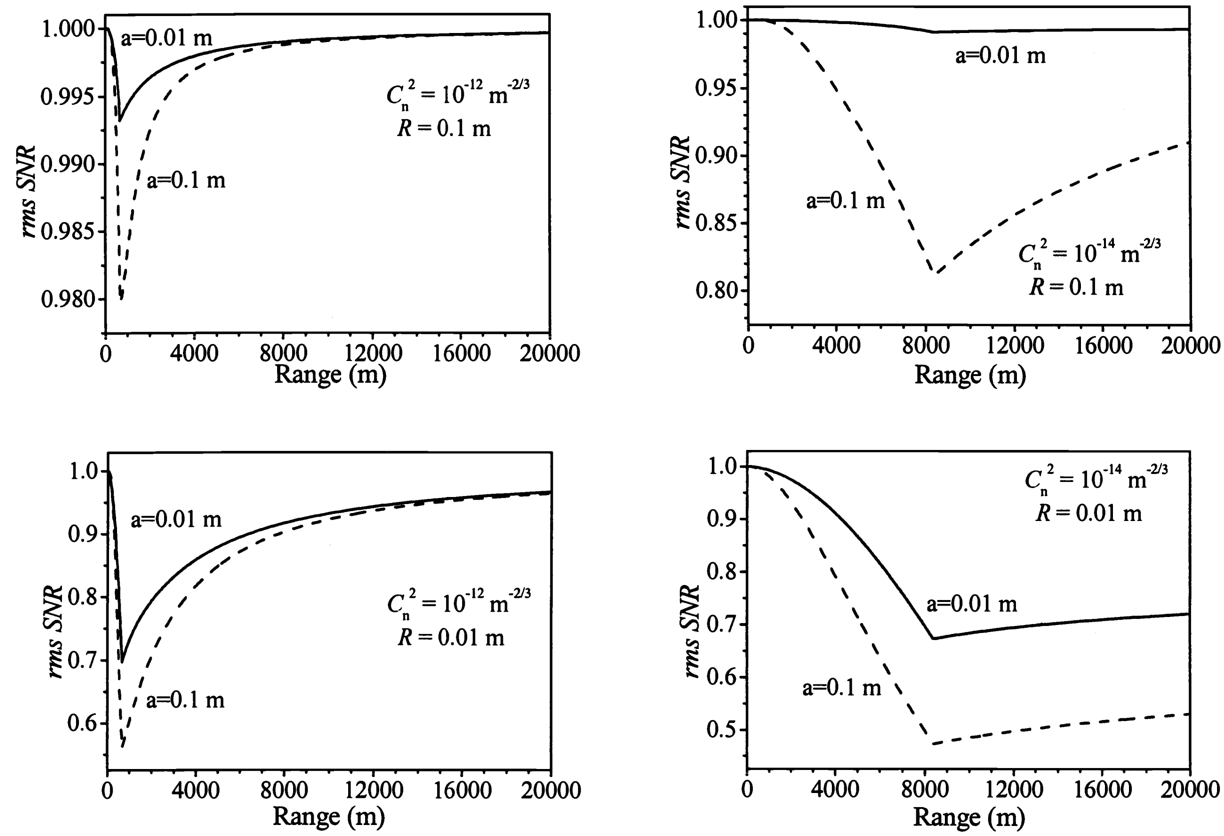

Figure 1: *rmsSNR* as a function of the range along the line of sight for $\lambda = 10.6$ μm.

4. CONCLUSION

A clear and compact expression is derived heuristically of the *rmsSNR* at coherent heterodyne detection of lidar returns. On the basis of the expression derived, an effect is revealed of aperture averaging of the "extraspeckle" intensity fluctuations of the detected optical signals. It is shown that at conventionally large receiving aperture radii the "extraspeckle averaging" effect leads to "speckle-like" statistical behaviour of the coherent lidar signals, independently of the statistics of the lidar return intensity. Thus, this effect should seemingly be taken into account when interpreting experimental results[5,6] concerning the statistical properties of the coherent-lidar signal power.

A clearer interpretation (understanding) of the speckle propagation through turbulent atmosphere is necessary based on a simplified, but physically viewable theory.

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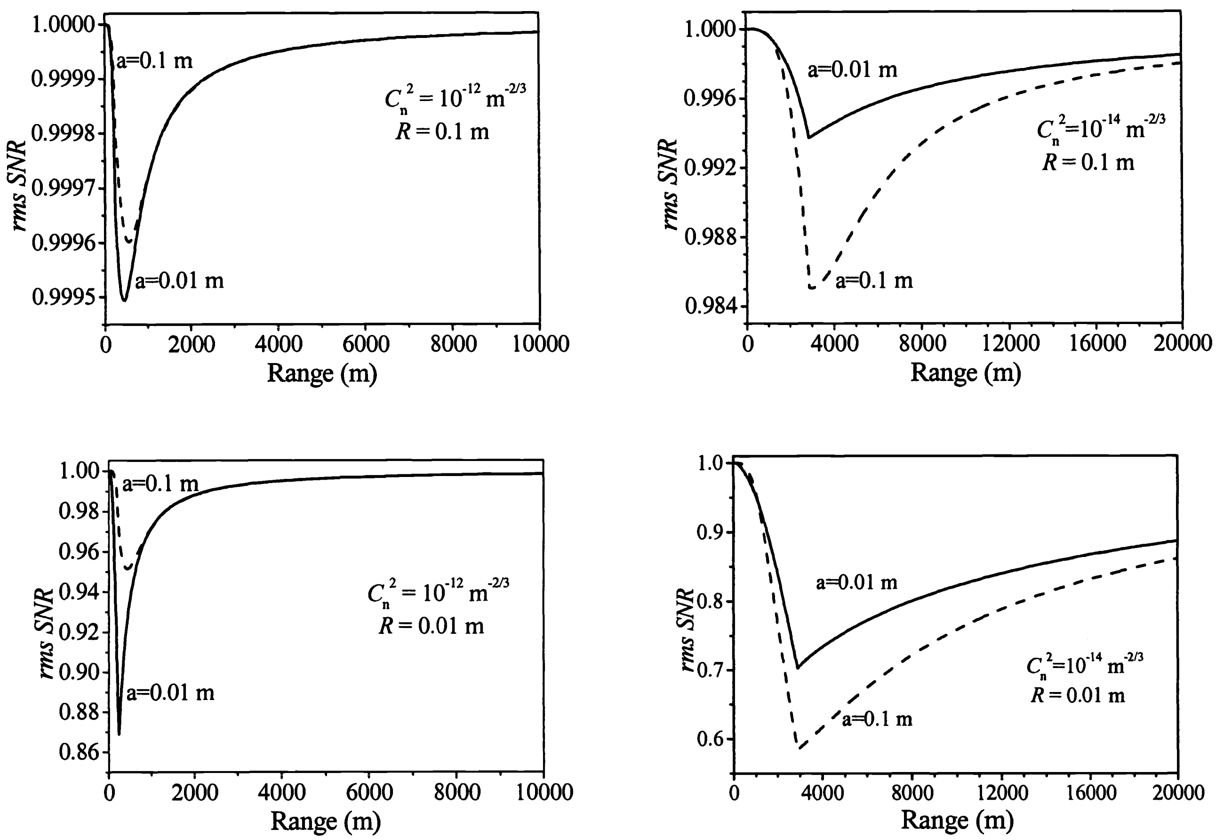

Figure 2: *rmsSNR* as a function of the range along the line of sight for $\lambda = 2$ μm.

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