On the determination by coherent lidar of Doppler-velocity profiles in turbulent atmosphere

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ABSTRACT

The influence is investigated quantitatively of the velocity fluctuations in turbulent atmosphere on the formation of the autocovariance of coherent heterodyne aerosol lidar signals. A multishot, high pulse repetition rate lidar operation is supposed. The limit cases of long-term and short-term averaging are especially considered, when the observation (data accumulation) time is respectively much larger or much less than the correlation time of the fluctuation process. As a result, the intuitive conception is proved and illustrated quantitatively .that a long-term averaging, under stationary conditions, allows one to obtain (on the basis of the autocovariance) a range-resolved estimate of the parent population mean Doppler-velocity profile; a short-term averaging allows one to determine a (near) instantaneous range-resolved Doopler-velocity profile.

Keywords: Coherent heterodyne detection, coherent lidar, turbulent atmosphere

1. INTRODUCTION

The autocovariance of the coherent heterodyne lidar signal is a basis for determination of range-resolved Doppler-velocity profiles in the atmosphere[1,2]. Averaging over some sufficient number N of conjecturally independent signal realizations (obtained by N laser shots) is a way to obtain accurate autocovariance estimates. For a long-enough observation (data acumulation) time exceeding essentially all the temporal correlation scales of the signal, the fluctuations of all types would be averaged. Then, assuming statistically stationary atmospheric conditions, the autocovariance estimate would be a basis for the determination of a (long-term average) estimate of the mean range-resolved parent-population Doppler-velocity profile[1,2].

The contemporary (powerful-enough) pulsed laser transmitters for coherent lidars can have a pulse repetition rate of the order of one kHz[3]. Consequently, for a few seconds they can provide a sufficiently large number of signal realizations such that the small temporal scale signal fluctuations, due e.g. to the reflective-speckle and refractive-turbulence effects as well as to the (weak) fast varying [small temporal (and spatial) scale] turbulent velocity fluctuations, to be averaged[4,5]. At the same time the stronger, slowly varying [large temporal (and spatial) scale] turbulent velocity fluctuations, whose correlation scales exceed the observation time, will not be averaged. They should take part in the formation of a near instantaneous (short-term average) range-resolved Doppler-velocity profile affecting (and obtainable from) the autocovariance estimate.

The main purpose of the present study is to substantiate and illustrate quantitatively the above-described qualitative conception about the practical formation of the autocovariance of coherent aerosol lidar signals. For this purpose a multishot, high pulse repetition rate lidar sensing is analyzed. The limit cases of long-term and short-term averaging are especially considered, when the observation time is respectively much larger or much less than the mean correlation time of the turbulent velocity fluctuation process. As a result, the intuitive (qualitative) conception is proved and illustrated quantitatively that a long-term averaging, under stationary conditions, allows one to obtain (on the basis of the autocovariance) a range-resolved estimate of the parent-population mean Doppler-velocity profile; a short-term averaging allows one to determine a (near) instantaneous range-resolved Doppler-velocity profile.

2. HETERODYNE SIGNAL AUTOCOVARIANCE

Since the coherent lidar return signal $I(t)$ is in general a nonstationary random process, its autocovariance $Cov(t,\theta)= \langle I^*(t)I(t+\theta)\rangle$ depends not only on the time shift θ but on the moment t as well. It is assumed here that t is the time

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after the pulse emission that corresponds to a position $z=ct/2$ of the pulse front along the lidar line of sight. The above-employed designations * and <.> denote respectively complex conjugation and ensemble average. In the case of multishot lidar operation we shall consider here, the autocovariance estimate $\hat{Cov}(t,\theta)$ has usually been obtained on the basis of the relation

$$\hat{\mathrm{Cov}}(t,\theta) = N^{-1}\sum_{k=1}^{N} I_k^*(t) I_k(t+\theta)\,, \qquad (1)$$

where $I_k(t)$ (k=1,2,...N) are N statistically independent signal realizations obtained by N laser shots. The pulse repetition rate r is supposed to be high-enough to ensure, for an observation time T, a sufficiently large value of $N=rT$ that enables one to average effectively most of the random factors disturbing $I_k(t)$ and respectively $\hat{Cov}(t,\theta)$. Such factors are, for instance, the speckle fluctuations (with temporal scales ~ 1 μs), the frequency and phase fluctuations in the sensing laser pulses, the turbulence-due amplitude and phase fluctuations (with temporal scales ~ 1 ms), etc. For a conventional observation time, of the order of a few seconds, the above-mentioned random factors would be averaged at a value of r ~ 200-300 Hz to 1 kHz. The turbulent velocity fluctuations however might not be averaged because of their large mean temporal scale t_c that depends on the outer spatial turbulence scale L_o and the translational air velocity V_T ($t_c \sim L_o/V_T$). They will be entirely averaged only when $T \gg t_c$. In the case of an overfilled observation period we shall suppose (when $N > T/t_c$), the averaging efficiency is determined by the relation between T and t_c. Under stationary conditions, the full-averaging procedure leads to an estimate of the parent-population signal autocovariance $Cov(t,\theta)$. A detailed analytical expression of $Cov(t,\theta)$ is derived in Ref.2. A concise writing of this expression, suitable for the further analysis, is

$$\mathrm{Cov}(t,\theta) = \int_0^{ct/2} dz' \Phi[t,\theta,\omega_m(z'),z']\gamma(z',2\omega_o\theta/c)\ , \qquad (2)$$

where Φ is a function of the corresponding variables, c is the speed of light, j is imaginary unity, $\omega_0 = 2\pi c/\lambda$ is the circular frequency of the sensing radiation and λ is its wavelength, $\omega_m(z') = \omega_0\chi(z') - \omega_h$, ω_h is the circular frequency of the local oscillator, $\chi(z') = 1-2\mathrm{v}(z')/c$, $\mathrm{v}(z')$ is a range-resolved parent-population mean radial (Doppler) velocity profile, and $\gamma(z',y) = \int \exp[-jy\tilde{\mathrm{v}}(z')]p[\tilde{\mathrm{v}}(z')]d\tilde{\mathrm{v}}$ is the characteristic function corresponding to the probability density distribution $p[\tilde{\mathrm{v}}(z')]$ of the radial-velocity fluctuations $\tilde{\mathrm{v}}(z')$ ($\langle\tilde{\mathrm{v}}(z')\rangle = 0$). There are some inverse algorithms[1,2], developed on the basis of the expression of Eq.(2), for retrieving the mean range-resolved Doppler velocity profile $\mathrm{v}(z')$ at a known estimate of $Cov(t,\theta)$. These algorithms, as well as Eq.(2) itself, are strictly valid when $T \gg t_c$. Is however Eq.(2) of use when $T \ll t_c$ or $T \sim t_c$, and what kind of results would be obtained in this case by the retrieving algorithms? For $T < t_c$ or $T \sim t_c$ the turbulent velocity fluctuations $\tilde{\mathrm{v}}(z')$ cannot be fully averaged. Then, instead of $\gamma(z',2\omega_0\theta/c)$, Eq.(1) will contain, as an integrand factor, another quantity

$$J(z',l,T) = N^{-1}\sum_{k=1}^{N} \exp(il\tilde{\mathrm{v}}_k) + o(N^{-1/2})\ , \qquad (3)$$

where $l = 2\omega_0\theta/c$, $\tilde{\mathrm{v}}_k = \tilde{\mathrm{v}}(k\Delta\tau, z')$ is the turbulent velocity fluctuation at the k-th laser shot at a distance z' along the line of sight, $\Delta\tau$ is the time interval between two adjacent laser shots, and $T=N\Delta\tau$ is the measurement (observation0 time. Since $\tilde{\mathrm{v}}$ does not change essentially within time intervals of the order of $\Delta\tau$ (when e.g. $\Delta\tau \sim$ 1ms and $V_T \sim$ 10-20 m/s), instead of Eq.(3), one can write that

$$J(z',l,T) = T^{-1}\int_0^T \exp[il\tilde{\mathrm{v}}(\tau,z')]d\tau + o(T^{-1/2})\ . \qquad (4)$$

The statistical behavior investigated below of the random quantity $J(l,T)$ (z' is omitted in what follows) outlines the areas and the sense of validity of Eq.(2) as well as its validity in general.

3. STATISTICAL CHARACTERISTICS OF *J(L,T)*

The temporal turbulent velocity fluctuations $\tilde{\mathrm{v}}(\mathrm{t}')$ may be considered as jointly Gaussian ones with a correlation coefficient

$$K(\tau) = \langle\tilde{\mathrm{v}}(t')\tilde{\mathrm{v}}(t'+\tau)\rangle / \langle\tilde{\mathrm{v}}^2(t')\rangle = F(\tau)/F(0)\ , \qquad (5)$$

where $F(\tau)=\int_0^\infty dK\cos(KV_T\tau)(K^2+K_o^2)^{-5/6}\exp(-K^2/K_m^2)$ (see e.g. Ref.2), $K_o=1/L_o$, $K_m=5.92/l_m$, $l_m=l_o(15C^2)^{3/4}$, $C^2=1.77$, and l_o is the inner spatial turbulence scale. However, to avoid computational difficulties and obtain more viewable results we shall use a simpler approximation of $K(\tau)$, namely

$$K(\tau)\approx\exp\{-|K_oV_T\tau|\}\quad, \tag{6}$$

where |.| denotes module.

On the basis of Eq.(4), taking into account the jointly-Gaussian statistics of $\tilde{v}$, we obtain the following statistical characteristics of $J(l,T)$:

$$\langle J(l,T)\rangle_{\tilde{v}_1}=\exp(-\sigma_v^2 l^2/2)T^{-1}\int_0^T d\tau\exp\{iK(\tau)l\tilde{v}_1+\sigma_v^2 l^2K^2(\tau)/2\} \tag{7}$$

(conditional ensemble-mean value, under condition that $\tilde{v}=\tilde{v}_1$ at the first laser shot), and

$$\sigma_{n\tilde{v}_1}=\left\{\left\langle|J(l,T)|^2\right\rangle_{\tilde{v}_1}-\left|\langle J(l,T)\rangle_{\tilde{v}_1}\right|^2\right\}^{1/2}/\left|\langle J(l,T)\rangle_{\tilde{v}_1}\right| \tag{8}$$

(conditional normalized standard deviation),

where $\sigma_v^2=<\tilde{v}^2>$, and $\left\langle|J(l,T)|^2\right\rangle_{\tilde{v}_1}=T^{-2}\exp(-l^2\sigma_v^2)\int\int_0^T dtdt'\,\exp\{il\tilde{v}_1[K(t)-K(t')]+\sigma_v^2 l^2[K(t-t')+[K(t)-K(t')]^2/2]\}$. The analysis of Eqs.(7) and (8) shows that $\langle J(l,T)\rangle_{\tilde{v}_1}\to\exp(il\tilde{v}_1)$ at $T\to 0$ [$T<<t_c=(K_oV_T)^{-1}$], and $\langle J(l,T)\rangle_{\tilde{v}_1}\to\exp(-\sigma_v^2l^2/2)$ at $T\to\infty$ ($T>>t_c$); in both the limiting cases $\sigma_{n\tilde{v}_1}\to 0$. Consequently, in these cases (when $T<<t_c$ and $T>>t_c$) the values of $J(l,T)$ obtained on the basis of experimental data are good estimates of the corresponding conditional ensemble-mean values $\langle J(l,T)\rangle_{\tilde{v}_1}$; the less the value of l the better the estimate of $\langle J(l,T)\rangle_{\tilde{v}_1}$ (the less the value of $\sigma_{n\tilde{v}_1}$). Therefore one can replace $J(l,T)$ [$\gamma(l,T)$] in Eq.(2) by $\exp(il\tilde{v}_1)$ in the former case, and by $\exp(-\sigma_v^2l^2/2)$, in the latter case. Thus, at $T<<t_c$ Eq.(2) describes a natural situation where the mean Doppler velocity profile for the period T is equal to $v(z')+\tilde{v}_1(z')$, i.e. to an instantaneous range-resolved Doppler velocity profile that can be determined by use of high-range-resolution retrieving algorithms[1,2]; certainly, in this case the velocity fluctuations around $v+\tilde{v}_1$ turn out to be negligible. At $T>>t_c$ Eq.(2) describes as expected the parent-population average signal autocovariance characterized by the parent-population mean Doppler velocity profile $v(z')$ (obtainable by use of high-resolution retrieving algorithms) and by the full-scale velocity variance σ_v^2; in this case $\exp(-\sigma_v^2l^2/2)$ is in fact the expression of the characteristic function $\gamma(l)$ corresponding to Gaussian-distributed velocity fluctuations.

Results from detailed numerical calculations based on Eqs.(4)-(8) are given in the figures below. They are in accordance with the above-mentioned general analytical conclusions and illustrate the intermediate case, when $T\sim t_c$. In

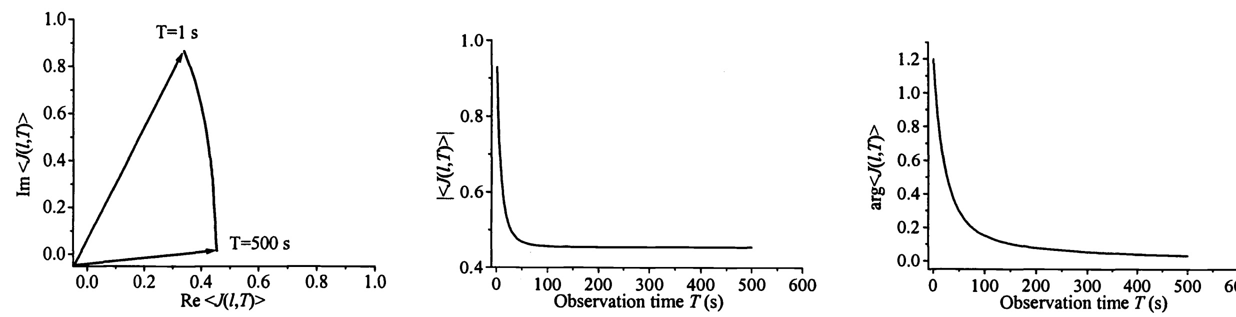

Figure 1: Conditional ensemble-mean value of $J(l,T)$ under condition that $\tilde{v}_{initial}=\tilde{v}_1=2$ m/s; $\lambda=2$ μm, $L_o=100$ m, $V_T=10$ m/s, $\sigma_v=2$ m/s, $\theta=100$ ns, $l=0.628$ s/m.

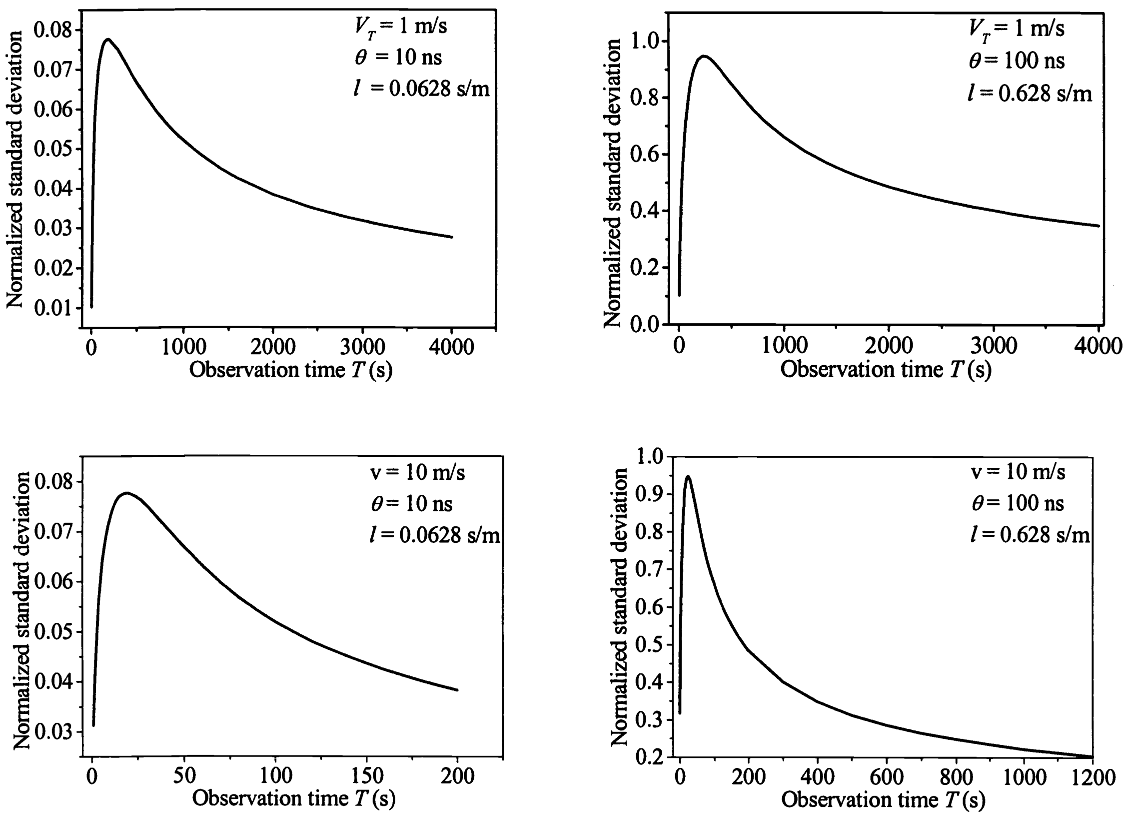

Figure 2: Conditional normalized standard deviation of $J(l,T)$ for $\lambda = 2$ µm.

this case $\arg\langle J(l,T)\rangle_{\tilde{v}_1} \neq 0$ and describes a mean (for the measurement interval [0,T]) range-resolved contribution to the Doppler velocity profile v(z'). At the same time $\exp(-\sigma_v^2 l^2/2) < \left|\langle J(l,T)\rangle_{\tilde{v}_1}\right| < 1$, i.e. the effective velocity variance with respect to v(z') is less than σ_v. Let us also note that in the intermediate case $\sigma_{n\tilde{v}_1}$ reaches a maximum $\sigma_{nm\tilde{v}_1}$ for some value of T~t_c. That is, here $\langle J(l,T)\rangle_{\tilde{v}_1}$ is a worse estimate of $J(l,T)$ than in the limiting cases $T \ll t_c$ and $T \gg t_c$. The value of $\sigma_{nm\tilde{v}_1}$ (as of $\sigma_{n\tilde{v}_1}$ in general) increases with l (with λ^{-1} and θ).

4. CONCLUSION

Eq.(2) describes adequately the coherent heterodyne-signal autocovariance at arbitrarily-long measurement times T. Thus, this equation is an useful basis for deriving inverse algorithms for retrieving range-resolved Doppler velocity profiles (along the lidar line of sight) at known (experimentally determined) estimates of the signal autocovariance. The duration T of the measurement (observation) interval conditions the type of the retrieved profiles.

A long-term observation procedure ($T \gg t_c$) allows one to retrieve (the autocovariance contains) a range-resolved estimate of the parent-population mean Doppler velocity profile at a full-scale velocity fluctuation variance.

A short-term measurement procedure ($T \ll t_c$) enables one to restore (the autocovariance contains) a near instantaneuos range-resolved Doppler velocity profile at a negligible velocity fluctuation variance.

A "middle-term" observation procedure allows one to obtain an estimate of a mean, for the observation period, range-resolved Doppler velocity profile with effective velocity fluctuation (around the mean profile) whose variance is less than the parent-population (full-scale) velocity variance.

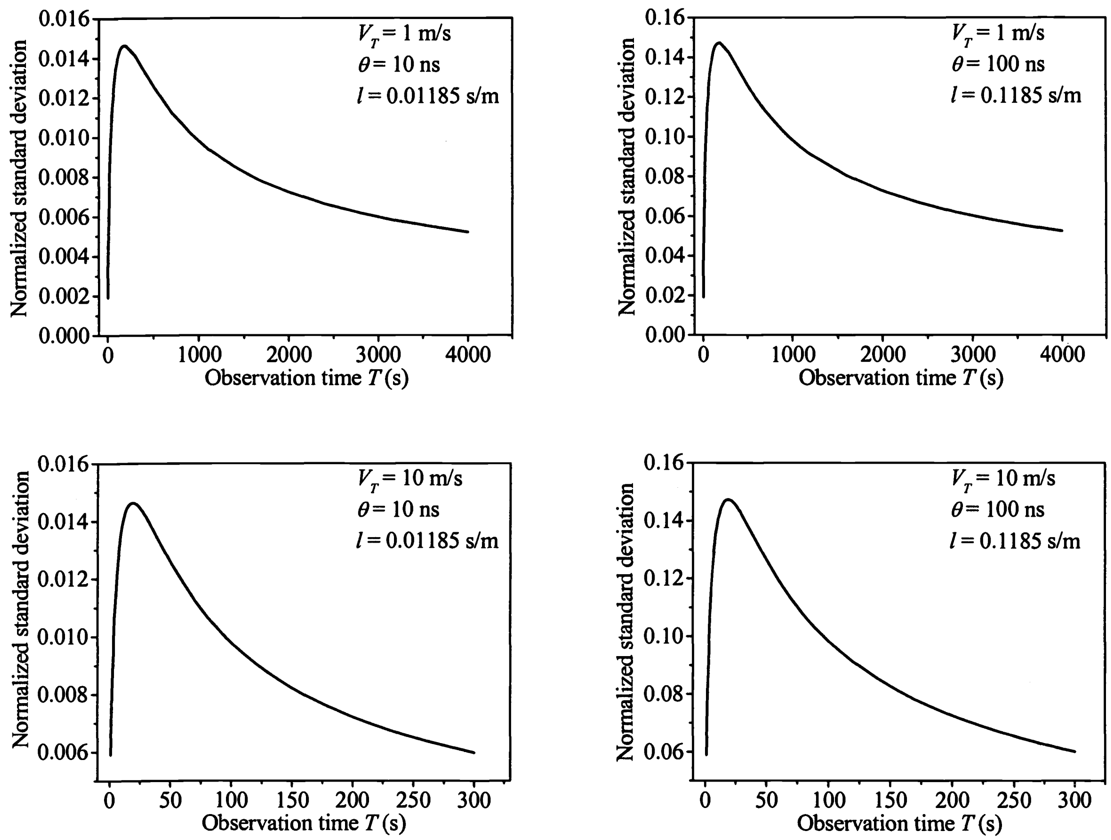

Figure 3: Conditional normalized standard deviation of $J(l,T)$ for $\lambda = 10.6$ μm.

As a whole, a general validity of Eq.(2) is established and substantiated, independently of the duration of a multishot lidar sensing. The duration of a multishot coherent lidar operation is only a time interval of averaging over. In all cases, the restored range-resolved Doppler velocity profiles are average ones over the corresponding measurement intervals.

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