Monte-Carlo Analysis of the Accuracy of a Novel Thomson Scattering Lidar Approach for Determination of the Electron Temperature in Fusion Plasmas

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Abstract. The accuracy of a novel Thomson scattering lidar approach for determination of the electron temperature in fusion plasmas is investigated by statistical Monte-Carlo modeling of the measurement process. The approach is based on the unambiguous temperature dependence of the ratio of the signal powers of two wide Thomson scattering spectral regions. It is shown that the accuracy of the novel approach is comparable with those of the center-of-mass wavelength approach we have developed recently and the commonly used fitting approach. Thus, such an approach would be advantageous by the possibility of using essentially simpler hardware consisting of only two receiving spectral channels.

Keywords: Fusion plasma diagnostic, Thomson scattering LIDAR, Electron temperature profiles
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INTRODUCTION

The knowledge of the electron temperature and density profiles in the tokamak plasmas is very important for understanding, characterizing and controlling the plasma processes. Therefore, these parameters need to be measured with a good spatial and temporal resolution. Because of the high characteristic temperatures in thermonuclear plasmas, from about 200 eV (~2x10^6 K) near the edge of the torus to 10-20 keV [~(1-2.3)x10^8 K] in the center, the only effective methods for sensing them turn out to be the contactless passive and active optical or microwave ones. Among these, the lidar Thomson scattering diagnostic [1] is especially appropriate for simultaneous determination of temperature and density profiles in fusion plasmas. It is based on the remote sensing of the plasma with an intense laser pulse and on the detection of the backscattered light from the plasma electrons. This diagnostic is in operation at the JET tokamak now and is intended to be used in ITER.

The approach used so far for electron temperature and pressure determination is based on log-linear or non-linear fit of the experimentally-obtained lidar-return spectra to the corresponding theoretical expression. Recently we proposed and developed two novel approaches for determination of electron temperature T_e by analyzing the relativistic Thomson scattering (TS) spectrum [2,3]. The first one is based on the unambiguous dependence of the "center-of-mass wavelength" of the lidar-return spectrum on T_e while the second one uses the unambiguous temperature dependence of the ratio of the signal powers of two spectral regions. The potential accuracy of the proposed methods is estimated on the basis of the derived analytical expressions of the corresponding rms relative errors in the determination of T_e [2].

The main goal of the present work is to analyze in detail the error in the determination of the electron temperature in tokamak plasmas by the approach based on the ratio of the signals from two wide spectral intervals, depending on the temperature itself and the signal-to-noise ratio achieved. For this purpose Monte-Carlo simulations of the temperature measurement process have been performed taking into account the photon flux measurement specificity, the bremsstrahlung contribution to the plasma light background and the Poisson noise effects. The numerical analysis shows that after an optimization the approach based on the ratio of the signals from two spectral intervals would have comparable accuracy with the other two approaches. Also, the possibility of using essentially

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simpler hardware consisting of only two receiving spectral channels could be an additional practical advantage of this method.

In Section 2 we describe the Thomson backscattering signal from fusion plasmas and the relevant plasma background light. The proposed approach based on the ratio of the signals from two spectral intervals is described in Section 3, and the results from Monte Carlo simulations are presented in Section 4. The main results are summarized in Section 5.

LIDAR SCATTERING SIGNAL AND PLASMA LIGHT BACKGROUND

The Thomson backscattered signal detected by a lidar is quantitatively analyzed by using the lidar equation. It describes the relation between the measured profile of the received backscattered light power, the parameters of the lidar system and the characteristics of the investigated high-temperature plasma. As lidar is a time-of-flight technique, the received power at the moment $t=t(z)$ after the pulse emission can be expressed as a function of the range $z=z(t)$, i.e. $t(z)$ is an unambiguous linear function of the line-of-sight (LOS) coordinate (distance) of the corresponding scattering volume. In the following analysis we will consider the spectral form of the lidar equation. The received power of the backscattered radiation at a wavelength λ_s could be written in the form $P[\lambda_s;t(z)]=Nh\nu_s$, where $N=N[\lambda_s;t(z)]$ is the photoelectron detection rate, h is the Planck's constant, $\nu_s=c/\lambda_s$, and c is the speed of light. Then, the lidar equation describing the photoelectron detection rate for the spectral interval $[\lambda_s, \lambda_s+d\lambda_s]$ has the form

$$N[\lambda_s;t(z)]d\lambda_s = \frac{N_0 c}{2} K_n(\lambda_i,\lambda_s)\Delta\Omega(z)\eta(\lambda_s,z)\beta[\lambda_i,\lambda_s,n_e(z),T_e(z)]\frac{\lambda_s}{\lambda_i}d\lambda_s \ , \qquad (1)$$

where N_0 is the number of photons in the sensing laser pulse, $K_n(\lambda_i,\lambda_s)=K_t(\lambda_i)K_t(\lambda_s)K_f(\lambda_s)EQE(\lambda_s)$, $K_t(\lambda_i)$, $K_t(\lambda_s)$, $K_f(\lambda_s)$ and $EQE(\lambda_s)$ are respectively the wavelength-dependent optical transmittance of the plasma-irradiating path, the optical transmittance of the scattered-light collecting path, the receiver filter spectral characteristic, and the effective quantum efficiency of the photon detection, $\Delta\Omega(z)$ is the solid angle of collection of the backscattered radiation as a function of the distance along the line of sight, $\eta(\lambda_s,z)$ is the lidar receiving efficiency, $n_e(z)$ is the electron concentration profile along the LOS, λ_i is the wavelength of the sensing laser radiation, $\beta[\lambda_i,\lambda_s,n_e(z),T_e(z)]$ is the Thomson backscattering coefficient (at an angle π) normalized by λ_i, at a distance z and a wavelength λ_s. The analytical expression for the Thomson backscattering spectrum for high-temperature relativistic plasma is given by [4,5]:

$$\begin{aligned}\beta[\lambda_i,\lambda_s,n_e(z),T_e(z)] = {} & \frac{n_e(z)r_0^2}{\sqrt{\pi}\lambda_i}\frac{c}{v_{th}(z)}\left(1+\frac{15}{16}\frac{v_{th}^2(z)}{c^2}+\frac{105}{512}\frac{v_{th}^4(z)}{c^4}\right)^{-1}\frac{(\lambda_i/\lambda_s)^4}{(1+\lambda_i/\lambda_s)} \\ & \times\ \exp\left\{-\frac{c^2}{v_{th}^2(z)}\left[(\lambda_i/\lambda_s)^{1/2}+(\lambda_s/\lambda_i)^{1/2}-2\right]\right\}q[\lambda_i,\lambda_s,T_e(z)]\end{aligned} \ , \qquad (2)$$

where r_0 is the classical electron radius, $v_{th}(z) =[2k_B T_e(z)/m_e]^{1/2}$ is the mean thermal velocity of the electrons, $T_e(z)$ is the electron temperature profile along the LOS, m_e is the electron rest mass, k_B is the Boltzmann constant, and $q[\lambda_i,\lambda_s,T_e(z)]$ is the depolarization term accounting for the relativistic depolarization effects on the backscattered radiation. For scattering at 180° the depolarization can be expressed in terms of exponential integral $E_n(p)$ [5,2]:

$$q[\lambda_i,\lambda_s,T_e(z)] = 1+2e^p\left[E_3(p)-3E_5(p)\right] = 1+\frac{1}{2}\left\{\frac{p^3}{2}-\frac{p^2}{2}-p-1\right\}+p^2\left(1-\frac{p^2}{4}\right)e^p E_1(p)\ , \qquad (3)$$

$$p=\frac{m_e c^2}{2k_B T_e(z)}\left(\sqrt{\lambda_s/\lambda_i}+\sqrt{\lambda_i/\lambda_s}\right), \text{ and } E_n(p)=\int_1^{\infty}\frac{e^{-px}}{x^n}dx\,.$$

If we consider M receiving spectral channels with wavelength intervals $[\lambda_{s1k},\lambda_{s2k}]$ (k=1,...,M), the mean return signal in each of them, in number of photoelectrons per second, could be written as

$$N_k[\lambda_{s1k},\lambda_{s2k},z(t)]=\frac{N_0 c}{2}\Delta\Omega[z(t)]\int_{\lambda_{s1k}}^{\lambda_{s2k}} K_n(\lambda_i,\lambda_s)\eta(\lambda_s,z(t))\beta[\lambda_i,\lambda_s,n_e[z(t)],T_e[z(t)]]\frac{\lambda_s}{\lambda_i}d\lambda_s \ . \quad (4)$$

The dominating component of the fusion plasma background light in the visible spectral range is the bremsstrahlung. The bremsstrahlung photon emissivity spectrum from fusion plasma per unit solid angle is given by the expression [6,7]:

$$\frac{dE}{d\Omega}=\frac{0.95\times10^{-19}}{\lambda\, 4\pi}n_{\mathrm{e}}{}^2(z)Z_{\mathrm{eff}}(z)[k_{\mathrm{B}}T_{\mathrm{e}}(z)]^{-1/2}\exp\left(-\frac{hc}{\lambda k_{\mathrm{B}}T_e(z)}\right)\tilde{g}_{ff}(\lambda,T_{\mathrm{e}}) \ , \quad (5)$$

where $Z_{\mathrm{eff}}(z)$ is the effective ion charge, the quantities $k_{\mathrm{B}}T_{\mathrm{e}}$ and hc/λ are in eV, $\exp[-hc/(\lambda\, k_{\mathrm{B}}T_{\mathrm{e}})]\approx1$ and $\tilde{g}_{ff}(\lambda,T_e)$ is the so-called Gaunt factor that depends weakly on T_{e} and on the radiation wavelength λ, and accounts for the quantum effects, the electron screening of nuclei, etc.[8]. For the photoelectron rate characterizing the parasitic background due to plasma light penetrating into the k-th spectral channel we obtain the following expression [2]:

$$N_{\mathrm{bk}}(\lambda_{\mathrm{s}1k},\lambda_{\mathrm{s}2k})=\frac{7.86\times10^{-20}}{4\pi}A_D\Delta\Omega_D\int_z dz n_{\mathrm{e}}^2(z)\left[k_{\mathrm{B}}T_{\mathrm{e}}(z)\right]^{-1/2}\int_{\lambda_{s1k}}^{\lambda_{s2k}} d\lambda_{\mathrm{s}}K_t(\lambda_{\mathrm{s}})K_f(\lambda_{\mathrm{s}})EQE(\lambda_{\mathrm{s}})\lambda_{\mathrm{s}}^{-1}\ln\left[k_{\mathrm{B}}T_{\mathrm{e}}(z)/(13.6h^2c^2/\lambda_s^2)^{1/3}\right] , \quad (6)$$

where A_D is the photon detector effective area and $\Delta\Omega_D$ is the solid angle determined by the relative aperture of the receiving optics.

TEMPERATURE MEASUREMENT APPROACH

Let us consider two spectral intervals of the relativistic Thomson backscattering spectrum from fusion plasma $[\lambda_{s11},\lambda_{s21}]$ and $[\lambda_{s12},\lambda_{s22}]$. According to Eq.(4), the ratio R of the signals is given as

$$R(T_e)=\int_{\lambda_{s11}}^{\lambda_{s21}} K_n(\lambda_i,\lambda_s)\eta(\lambda_s,z)\beta(\lambda_i,\lambda_s,v_{th})\frac{\lambda_s}{\lambda_i}d\lambda_s \Big/ \int_{\lambda_{s12}}^{\lambda_{s22}} K_n(\lambda_i,\lambda_s)\eta(\lambda_s,z)\beta(\lambda_i,\lambda_s,v_{th})\frac{\lambda_s}{\lambda_i}d\lambda_s \ . \quad (7)$$

This ratio is an unambiguous function of the electron temperature T_{e} and may be used for measuring it.

The linear error propagation approach to estimating the rms error δT_{e} in the determination of T_{e} [2] at relatively high signal-to-noise ratios $SNR_k=(N_k\tau_d)^{1/2}/(1+N_{bk}/N_k)^{1/2}$ leads to the following expression:

$$\delta T_{\mathrm{e}}=\left|d\ln R(T_{\mathrm{e}})/dT_{\mathrm{e}}\right|^{-1}\left\{\sum_{k=1}^{2}(N_k\tau_d)^{-1}(1+N_{\mathrm{b}k}/N_k)\right\}^{1/2} , \quad (8)$$

where τ_{d} is the system response time arising here as a factor accounting for the process of convolution between the lidar signal and the receiving system response shape.

NUMERICAL ANALYSIS OF THE THOMSON SCATTERING MEASUREMENTS

The goal of the numerical simulations performed was to analyze the error in the determination of the electron temperature by the approach based on the ratio of the signals from two spectral intervals. For this purpose the entire Thomson scattering measurement procedure in the plasma had to be simulated.

We suppose that the TS spectrum is observed within the wavelength region from λ_l=350 nm to λ_u =850 nm. This is in accordance with the spectral sensitivity ranges of the available photon detectors [9]. For applying the T_e – measurement approach discussed here, the wavelength interval $[\lambda_l,\lambda_u]$ is divided into two subintervals $[\lambda_l, \lambda_m]$ and $[\lambda_m,\lambda_u]$. The sensing laser is supposed to emit pulses of energy E_0=1 J at λ_i=694 nm. The minor radius of the torus at the LOS direction is supposed to be 1 m, i.e. the fusion plasma is supposed to occupy the LOS region of length of 2 m. The solid angle of collection is supposed to have a value of 0.005 sr at the center of the plasma cross-section. Further, we assume that the optical transmittance of the input path is $K_t(\lambda_i)$=0.75, the transmittance of the collecting path is $K_t(\lambda_s)$=0.25 for $\lambda_s \in [\lambda_l,\lambda_u]$, the receiver filter spectral characteristics $K_f(\lambda_s)$=1, and EQE=0.05 for both the measurement channels (see, e.g., [9]). The detector's etendue $E=A_D\Delta\Omega_D$ needed for the estimation of the plasma bremsstrahlung photoelectron rate is assumed to have a value of 1 cm^{2}sr. The factor of reducing the plasma light background conditioned by the plasma torus observation pupil is supposed to be 0.3. Finally, the system response time τ_d and the sampling interval τ_s of the analog-to-digital converter are assumed to be 500 ps and 100 ps, respectively. This choice corresponds to 7.5 cm spatial resolution interval and 1.5 cm sampling step, respectively.

In order to determine the calibration function $R(T_e)$, the electron temperature is varied from 0.1 keV to 10 keV (or to the maximum value we expect to be achieved in the plasma) with a step ΔT_e=0.1 keV. For these temperatures the corresponding mean TS photoelectron rates for both channels, $N_1(T_e)$ and $N_2(T_e)$, are evaluated on the basis of Eq.(4). Then the calibration function $R(T_e)=N_1(T_e)/N_2(T_e)$ is determined. The estimations performed show [2] that the rms error for T_e > 1 keV has minimum values at λ_m=600 nm. In Fig.1 we present the calibration function $R(T_e)$ obtained for this choice of the wavelength λ_m.

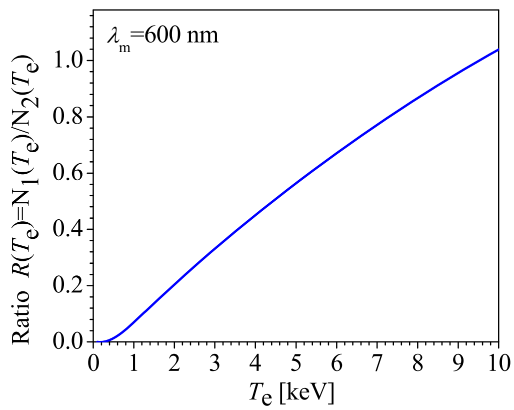

FIGURE 1. Calibration function $R(T_e)$ for the T_e - measurement approach based on the ratio of the signals from two spectral intervals.

In order to estimate the rms error in the determination of T_e as a function of T_e we performed Monte Carlo simulations at a fixed LOS position in the plasma. In this case the mean TS and background photoelectron rates [evaluated on the basis of Eqs.(4) and (6), respectively] are functions of T_e only. The temperature T_e is varied again with a step ΔT_e=0.1 keV. At known mean values $N_1(T_e)\tau_d$ and $N_2(T_e)\tau_d$, and $N_{br1}(T_e)\tau_d$ and $N_{br2}(T_e)\tau_d$, by using Poisson random-number generator we produce H realizations, $\hat{N}_1^l(T_e)\tau_d$, $\hat{N}_2^l(T_e)\tau_d$, $\hat{N}_{br1}^l(T_e)\tau_d$ and $\hat{N}_{br2}^l(T_e)\tau_d$, of the photoelectron counts of the TS signal and the background, l=1,…,H. Then we compose the quantities $\hat{N}_1^l\tau_d + \hat{N}_{br1}^l\tau_d - N_{br1}\tau_d$ and $\hat{N}_2^l\tau_d + \hat{N}_{br2}^l\tau_d - N_{br2}\tau_d$. At last, by using the reference functions $R(T_e)$ and the ratios $(\hat{N}_1^l\tau_d + \hat{N}_{br1}^l\tau_d - N_{br1}\tau_d)/(\hat{N}_2^l\tau_d + \hat{N}_{br2}^l\tau_d - N_{br2}\tau_d)$ we obtain H estimates $\hat{T}_e^l$ of T_e and an estimate $\hat{\delta T}_e$ of the rms error δT_e, given by

$$\hat{\delta T}_e = \left[H^{-1} \sum_{l=1}^{H} (\hat{T}_e^l - T_e)^2 \right]^{1/2} . \tag{9}$$

In Fig.2 the Monte-Carlo estimated relative rms error $\hat{\delta}T_e / T_e$ is compared with the theoretical estimate obtained by using Eq.(8) for the LOS position corresponding to the center of the plasma cross-section. The results presented are for two different values of the electron density $n_e(z)$=5x10^{19} m^{-3} (a) and 0.5x10^{19} m^{-3} (b) (in fact, for different SNR). It is seen that the error behavior is consistent with that predicted analytically. As can be expected, the error increases with the decrease of the n_e (i.e. with the decrease of SNR). Also, for a fixed value of the electron density, the SNR vary with the increase of the electron temperature. Naturally, the error increases with the decrease of the signal-to-noise ratios $\mathrm{SNR}_k=[N_k(T_e)\tau_d]^{1/2}/(1+ N_{\mathrm{br}k}(T_e)/N_k(T_e))^{1/2}$, k=1,2 [2]. For instance, at $n_e(z)$=5x10^{19} m^{-3} the lowest values of the relative error are achieved at T_e~5 keV, where SNRs have their maximum values: SNR_1~36 and SNR_2 ~ 50. At $n_e(z)$=0.5x10^{19} m^{-3} minimum values of the error are at T_e~4 keV, where SNR_1~12 and SNR_2 ~ 17. One may notice as well that the obtained by the simulations values of the error are greater than those predicted theoretically for T_e < 4 keV (a,b) and T_e > 12 keV (b) due to decreasing the values of SNR_k. As a whole, at $n_e(z)$=5x10^{19} m^{-3}, the relative error has values below 10 % for T_e > 1 keV, and even lower than 5 % for 3 keV< T_e <13 keV. In the next Fig.3 the recovered temperature values from different realizations (at T_e = 5 keV, $n_e(z)$ = 5x10^{19}m^{-3}) are shown.

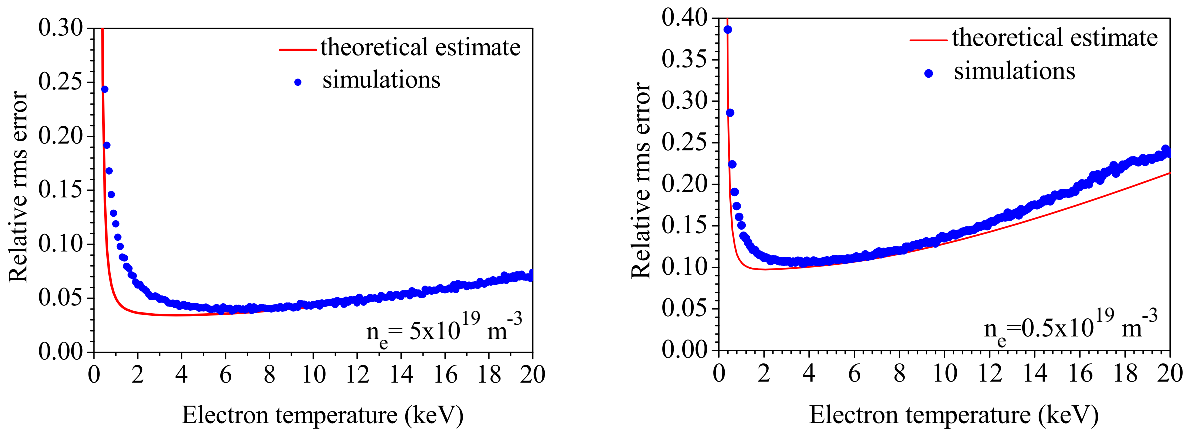

FIGURE 2. Relative rms error $\delta T_e/T_e$ in the determination of the electron temperature T_e for two different values of the electron density. The theoretical results and the results from Monte-Carlo simulations are represented respectively by curves and points.

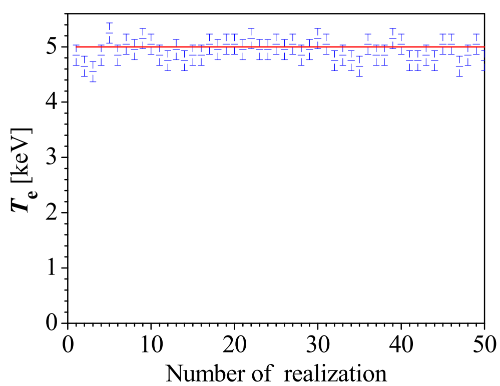

FIGURE 3. Recovered temperature values at the LOS position corresponding to the center of the plasma cross-section (at T_e = 5 keV, n_e=5x10^{19} m^{-3}) obtained from different realizations.

The comparison of the proposed method for determining the electron temperature with the widely used fitting approach and the developed by us center-of-mass wavelength approach [2,3] shows that they have comparable accuracy. This is illustrated in Fig.4, where the theoretically estimated relative errors of the three above mentioned approaches, for the same input parameters, as well as the corresponding results from the simulations are presented. The spectral intervals used in the center-of-mass wavelength and the fitting approaches are optimally chosen to ensure minimum measurement errors [2,3].

One restored LOS profile of the electron temperature is compared with the input model profile in Fig.5. In this case the fluctuating LOS profiles of the TS signal and plasma light background for each of the spectral channels are determined as above. The upper and lower curves denote the standard deviation limits in obtaining T_e. As seen, the procedure of recovering the T_e profile is stable and accurate.

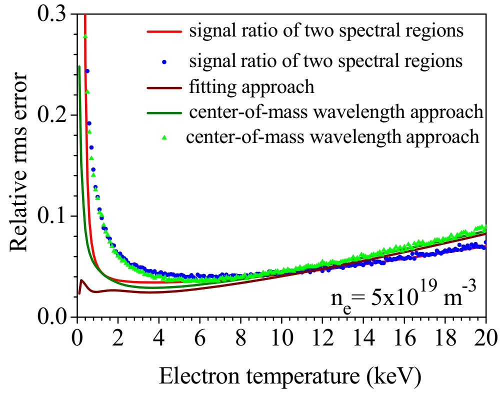

FIGURE 4. Theoretically estimated relative rms errors $\delta T_e/T_e$ in the determination of the electron temperature T_e by using the ratio of the signals from two spectral intervals, the center-of-mass wavelength, and the fitting approaches.

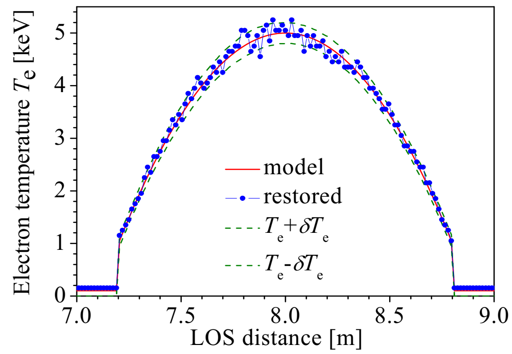

FIGURE 5. One restored LOS profile of the electron temperature T_e compared with the input model profile. The upper and lower curves denote the standard deviation limits in obtaining T_e.

CONCLUSION

In the present work we have estimated by numerical simulations the accuracy of the novel approach proposed for determination of the electron temperature profiles in fusion plasmas by Thomson scattering lidar. The approach is based on an analysis of the relativistic TS spectrum and uses the unambiguous temperature dependence of the ratio of the return-signal powers of two spectral regions. As a result of the performed Monte-Carlo simulations we have obtained the authentic rms temperature-measurement error as a function of the electron temperature and the signal-to-noise ratio. The results obtained confirm the analytical expressions in their region of validity, where the SNR is well above unity. The analysis of the results shows that at pulse energy of 1 J and sensing radiation wavelength of 694 nm the relative rms error is below 10% for temperatures between 1 keV and 20 keV, and even below 5 % for T_e between 5 keV and 13 keV and n_e = 5x10^{19}m^{-3}; in this case, the mean SNR achieved is about 40. Note that the relative rms error of the considered power-ratio approach at λ_m=600 nm is comparable with and even lower for T_e >11 keV than the errors intrinsic to the center-of-mass wavelength approach and the fitting approach. So, this approach may be advantageous due to the possibility of using essentially simpler hardware consisting of only two receiving spectral channels.

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REFERENCES

1. H. Salzmann et al., *Rev. Sci. Instrum.* **59**, 1451-1456 (1988).
2. L.L. Gurdev, T.N. Dreischuh, D.V. Stoyanov, "Potential accuracies of some new approaches for determination by Thomson scattering lidar of the electron temperature profiles in thermonuclear plasmas**",** *Proceedings of SPIE* **7027**, 702711 (2008).
3. T.N. Dreischuh et al., "Statistical modeling of the error in the determination of the electron temperature in JET by a novel Thomson scattering lidar approach", *Proc. 36th EPS Conference on Plasma Physics*, June 29-July 3, 2009, Sofia, Bulgaria.
4. M. Mattioli, "Incoherent light scattering from high temperature plasmas", EUR-CEA-FC-752 (1974).
5. O. Naito, H.Yoshida, T. Matoba, *Phys. Fluids* **B5**, 4256-4258 (1993).
6. J. Sheffield, *Plasma Scattering of Electromagnetic Radiation*, New York: Academic, 1975, pp. 191–210.
7. M. E. Foord, E. S. Marmar, J. L. Terry, *Rev. Sci. Instrum*. **53**, 1407-1409 (1982).
8. P.J. Brusaard, H.C. van de Hulst, *Rev. Mod. Phys*. **34**, 507-520 (1962).
9. R. Pasqualotto, P. Nielsen, *Rev. Sci. Instrum*. **75**, 3891-3893 (2004).